Review 24 Infinite series

don't confuse: sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \dots$ list

series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8} \dots$ sum of sequence

Tests for convergence / divergence

1. geometric $\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \dots$ look for common ratio

$\sum_{n=0}^{\infty} \frac{1}{2^n}$ converge to $\frac{a}{1-r}$ if $|r|<1$

diverge if $|r|>1$

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p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge if $p>1$

diverge if $p \le 1$

$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} \dots$ (harmonic if $p=1$)

$\sum \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} \dots$

other tests:

n^{th} term test: if seq $\not\to 0$, series diverges

what if seq $\to 0$, series ??

ratio test: factorials or n as an exponent

$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$ if $L<1$ series conv

$L>1$ series div

$L=1$ inconclusive

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Integral test

$\sum_{n=1}^{\infty} a_n$ & $\int_1^{\infty} a_n \, dn$ behave the same

comparison tests

direct comparison

if $0 \le a_n < b_n$ & $\sum b_n$ converges

then $\sum a_n$ converges

if $a_n \ge b_n$ & $\sum b_n$ diverges

then $\sum a_n$ diverges

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limit comparison $\lim_{n \to \infty} \frac{a_n}{b_n} = L$ $0 < L < \infty$

if a_n & b_n grow at the same rate

then $\sum a_n$ & $\sum b_n$ behave the same

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series with neg terms?

absolute convergence if $\sum |a_n|$ conv

then $\sum a_n$ conv

conditional conv. $\sum |a_n|$ div, $\sum a_n$ conv

↑ prove with AST

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Alternating Series Test

If ✓ 1. terms strictly alternate

✓ 2. magnitudes get smaller

$|a_{n+1}| < |a_n|$

✓ 3. $a_n \to 0$ (seq $\to 0$)

then the series converges

Remainder Theorem

Partial sum $= a_1 - a_2 + a_3 - a_4 + \dots a_n$

$\epsilon < |a_{n+1}|$

$a_{n+1} > 0$ underest.

$a_{n+1} < 0$ overest

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